Új Nemzeti
Kiválóság Program

VI. Nuclear Reactions

Lorant Csige
Laboratory for Nuclear Physics
Hungarian Academy of Sciences

Nuclear reactions

- Observables in nuclear reactions:
  - defined by external parameters: bombarding energy, potential barriers
  - defined by internal „parameters”: nuclear structure and reaction mechanism
- Basic reaction mechanisms: direct reaction and compound reaction
  - time evaluation is different
  - these are extremities: real reactions are mixed in nature

- Reaction channels:
  - $a + A \rightarrow (C^*) \rightarrow A + a$ elastic scattering
  - $a + A \rightarrow (C^*) \rightarrow A^* + a'$ inelastic scattering
  - $a + A \rightarrow (C^*) \rightarrow B + b$ A(a,b)B reaction
  - $a + A \rightarrow (C^*) \rightarrow C + \gamma$ radiative capture

incoming channel outgoing channel

- Direct reactions:
  - interaction time is short (10^{-22} s)
  - only a few nucleon is involved in the process
- Compound reactions:
  - interaction time is long (10^{-19}-10^{-16} s)
  - two-step process: forming a compound nucleus, then decay

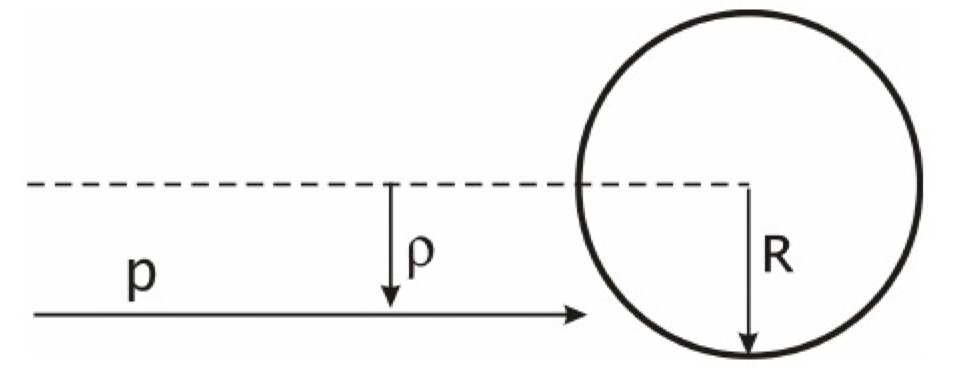
Conservation laws

- Electric charge
- Barion charge: total number of nucleons are conserved in the reactions
- Momentum conservation: in a fixed target experiment → $\mathbf{p}_a = \mathbf{p}_B + \mathbf{p}_b$
- Energy conservation: energy balance of nuclear reactions
  - $(m_a+m_A)c^2 + E_a+E_A = (m_b+M_B)c^2 + E_b + E_B$
  - reaction energy: $Q=(E_B+E_b)-(E_A+E_a)=(m_b+M_B)c^2-(m_a+m_A)c^2$
    - if $Q>0$ then reaction is exoerg (e.g. D+T→ ^{4}He + n → Q=17.6 MeV)
    - if $Q<0$ then reaction is endoerg (e.g. ^{14}N+^{4}He → ^{17}O + p → Q=-1.19 MeV), then the treshold energy is the laboratory system is: $E_{a,min}=Q(M_A+m_a)/M_A$

- angular momentum conservation:
  - $I_A+I_a+I_{Aa} = I_B+I_b+I_{Bb}$ where I_{Aa} and I_{Bb} are relative orbital angular momentum
  - if relative angular momentum is 0 → angular distribution of the reaction products has spherical symmetry in the center-of-mass system!

Centrifugal and Coulomb barrier

- Classic impact condition:
  - |**p**x**ρ**| ≤ pR = √(2mE)R
  - quantization: |**p**x**ρ**| → |lħ|
- Given E → $(l\hbar)^2 \leq 2mER^2$
  - limitation on l



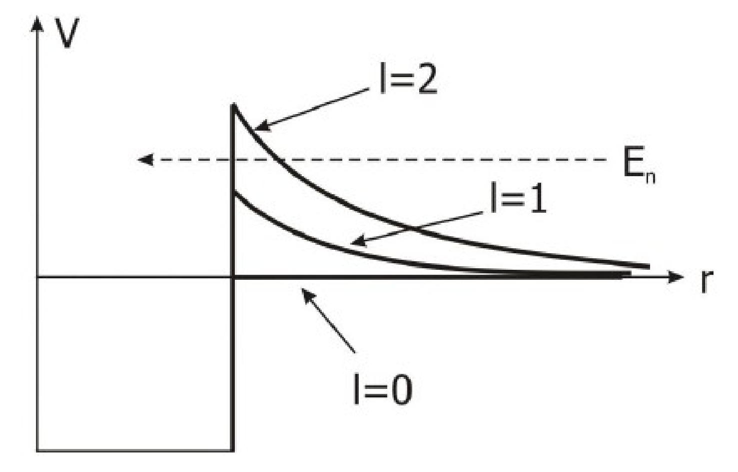
$$l_{max} = \frac{R}{\bar{\lambda}_n} \approx R(fm)\frac{\sqrt{E_n(MeV)}}{4.55}$$

for neutrons (for l=0, can interact with any E!!)

- Given **l** → $E \geqslant \frac{l^2\hbar^2}{2mR^2} \approx \frac{l(l+1)\hbar^2}{2mR^2}$

$$V_{cf} = \frac{l(l+1)\hbar^2}{2mr^2}$$

centrifugal potential (in QM)



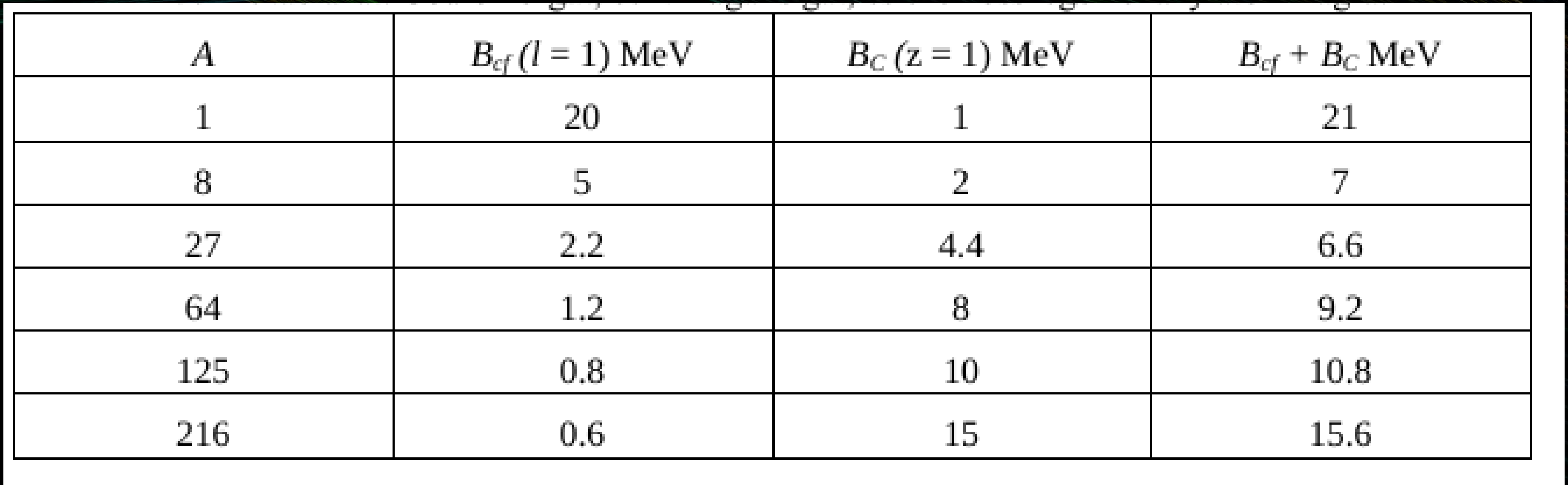
$$B_{cf}(MeV) = V_{cf}(r=R) = \frac{10\,l(l+1)\hbar^2}{A^{2/3}}$$

centrifugal barrier

Centrifugal and Coulomb barrier

- Coulomb barrier: in case of charged particles → centrifugal + coulomb

$$B_c(MeV) = V_c(r = R) = \frac{Zz}{A^{1/3}}$$



A	B_{cf} ($l = 1$) MeV	B_C ($z = 1$) MeV	$B_{cf} + B_C$ MeV
1	20	1	21
8	5	2	7
27	2.2	4.4	6.6
64	1.2	8	9.2
125	0.8	10	10.8
216	0.6	15	15.6

- Light nuclei → B_{cf} dominates; heavy nuclei → B_c dominates

Conservation laws (cont.)

- Parity is conserved in nuclear reactions (but not in beta-decay!):
  - $P_a P_A (-1)^{l_{aA}} = (P_{C*}) = P_b P_B (-1)^{l_{bB}}$
  - if C* has definite parity → the angular distribution of b and B in the center of mass system has backward-forward symmetry
  - in elastic scattering → $l_{bB} = l_{aA} \pm 2, \pm 4, \ldots$

- Isospin is conserved in nuclear reactions:
  - $T_a + T_A = (T_{C*}) = T_b + T_B$
  - special cases: $T_a=0$ and $T_b=0$ [e.g.: (α,α) (d,d) (α,d) reactions]

Neutron-induced reactions

- Neutron induced reactions are of great importance since only strong interaction is involved (no electromagnetic processes)

- (n,n) elastic scattering, Q=0:
  - neutron detectors and moderators in nuclear reactors

- (n,n’) inelastic scattering, Q<0

- (n,γ) radiative capture, Q>0:
  - cross section can be extremely high for low energy (and thermal) neutrons (<0.5 MeV) → ^{113}Cd rods and ^{135}Xe (fission product) poison

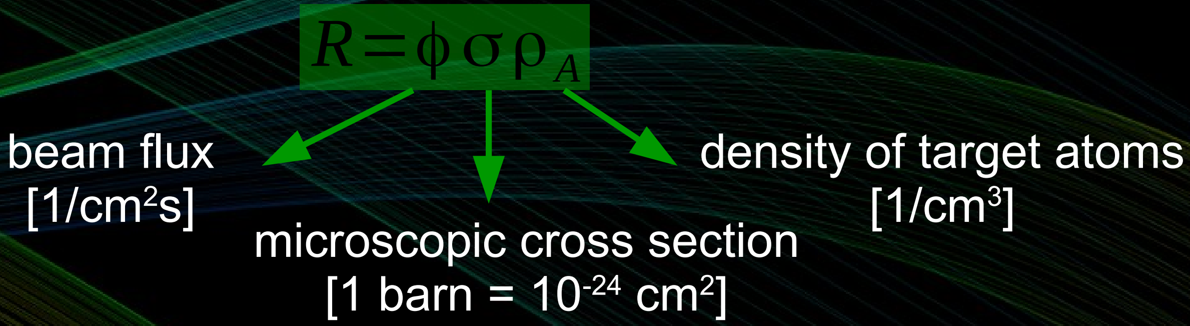
- (n,p) usually Q>0 since $m_n > m_p$:
  - in atmosphere: $^{14}N + n \rightarrow {}^{14}C + p$ (radiocarbon dating!)

- (n,α) and (n,2n) (n,3n): Q can be very high → neutron detection

- (n,f) neutron-induced fission, Q=200 MeV (for ^{238}U): see later….

Cross section of reactions

- nuclear reaction rate: number of reaction per unit time and volume



$$R = \phi \sigma \rho_A$$

beam flux
[1/cm^{2}s]

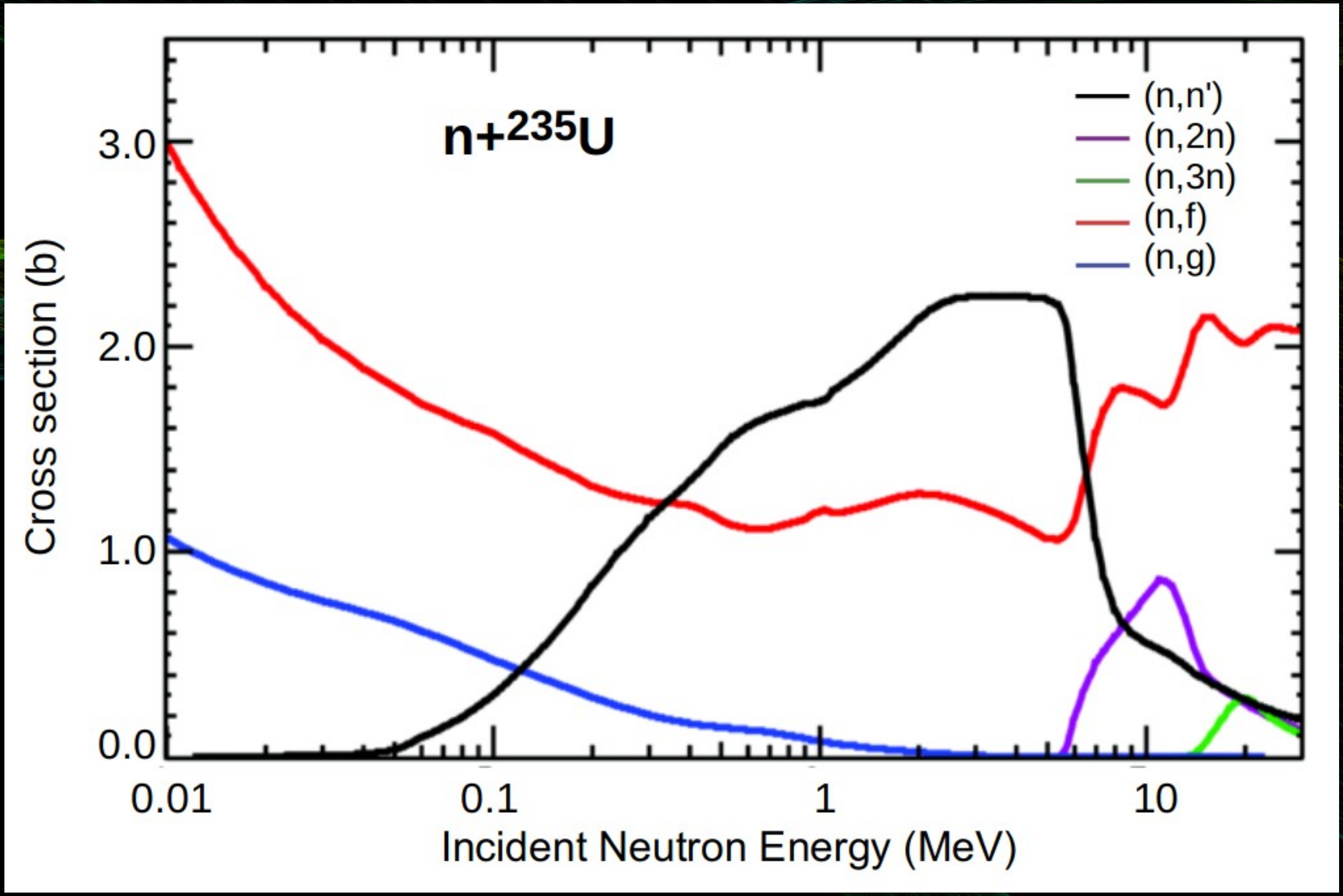
microscopic cross section
[1 barn = 10^{-24} cm^2]

density of target atoms
[1/cm^3]

- microscopic cross section is characteristic to the different reaction channels
- cross sections typically depends strongly on bombarding energy
- differential and integral cross sections: in the function of energy or/and angular distribution of the outgoing particles

Excitation functions

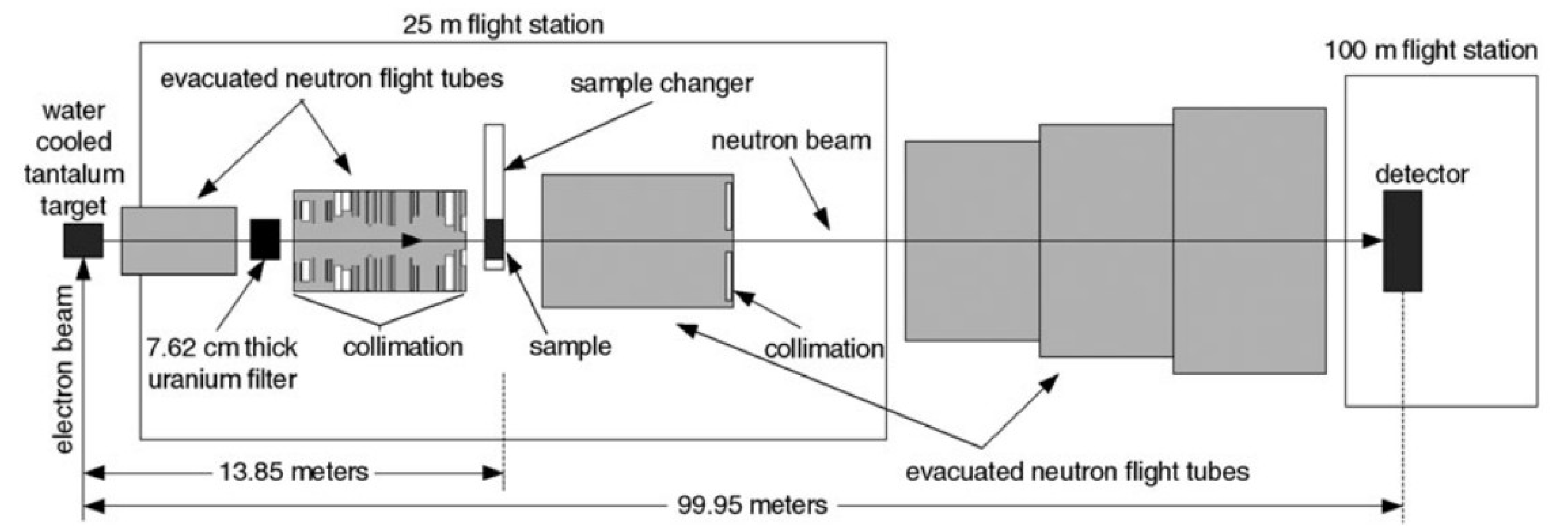
- Cross section of reactions depend on the bombarding energy: σ(E)



- neutron: at low energies the dependency is ~1/v=1/√E
- charged particle: Coulomb barrier → first increasing with E → after a maximum it is decreasing

Total neutron cross sections

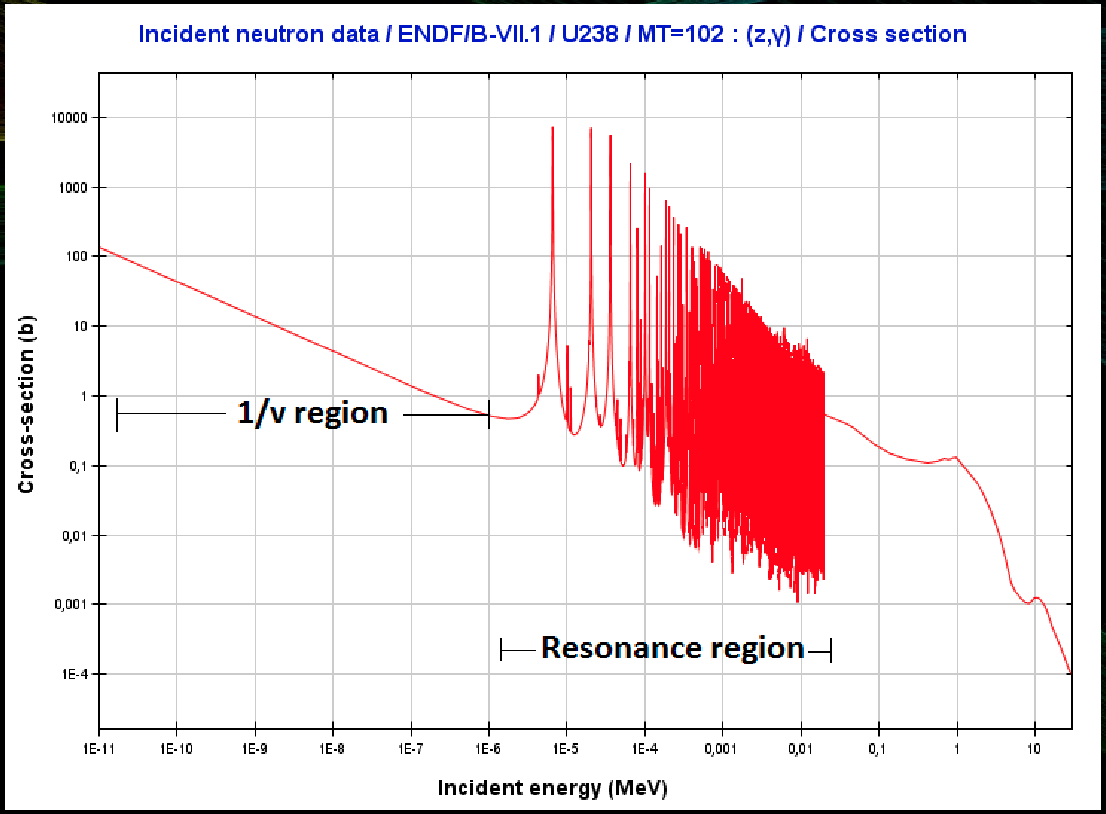
- All the possible processes are included: absorption and scattering
  - measuring without sample: I_0 and with sample: I the ratio I/I0 is determined
  - quite „simple” measurements



$$I = I_0 e^{-nl\sigma_T} \Rightarrow \sigma_T = \frac{1}{nl} \ln \frac{I_0}{I}$$

Total neutron cross sections

- First results: very narrow resonances in the excitation function $\sigma_T(E_n)$ were observed at low energies



- From τ= → $\tau=10^{-14}$ s which is long compared to the characteristic time of interaction (10^{-22} s~speed of nucleons within the nuclus)
- N. Bohr: compound nucleus model
  - kinetic energy + separation energy is spreading by collisions of nucleons → the incoming particle become indistinguishable from target nucleus → complete thermal equilibrium
  - quasi stationary state
- Decay of compund nucleus: by gamma emission (thermal energy) and by neutron emission (higher energy)

Total neutron cross section

- Cross section of compound nucleus formation is:

$$\sigma = \pi \bar{\lambda}^2 g \frac{\Gamma_n \Gamma}{(E - E_r)^2 + (\Gamma/2)^2}$$

E_r: resonance energy
g: statistical factor

$$g = \frac{2J+1}{(2I+1)(2i+1)}$$

multiplicity of the initial
and final state

- So the cross section of radiative capture is:

$$\sigma(n,\gamma) = \sigma \frac{\Gamma_\gamma}{\Gamma} = \pi \bar{\lambda}^2 g \frac{\Gamma_n \Gamma_\gamma}{(E - E_r)^2 + (\Gamma/2)^2}$$

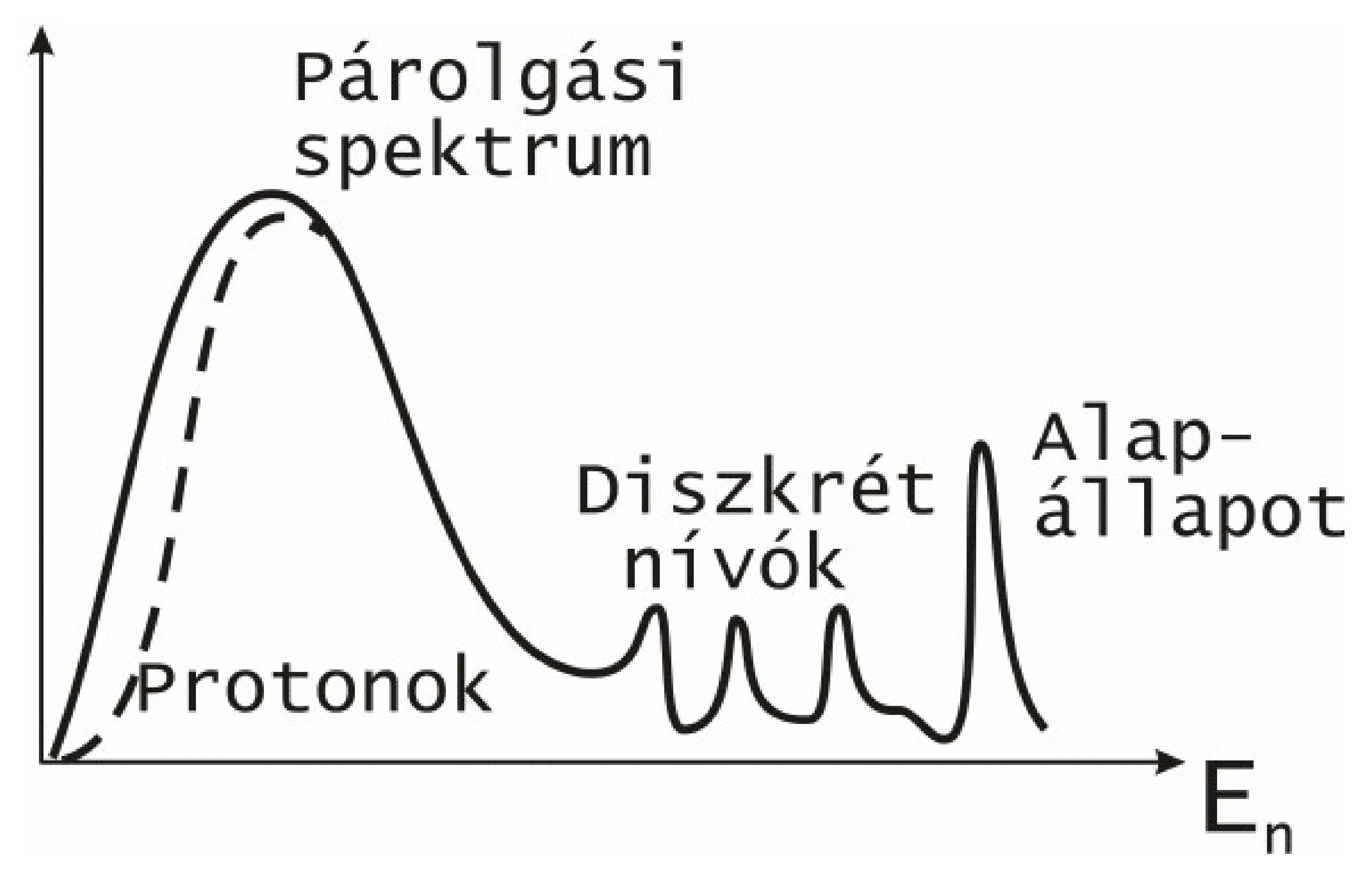
Breit-Wigner formula

→ if $E \ll E_r$ (and $E - E_r \gg \Gamma$) → $\sigma \sim 1/v$!!

- In the resonance region is $E_n < 1$ MeV

Total neutron cross section

- Maxwell evaporation spectrum



Optical model

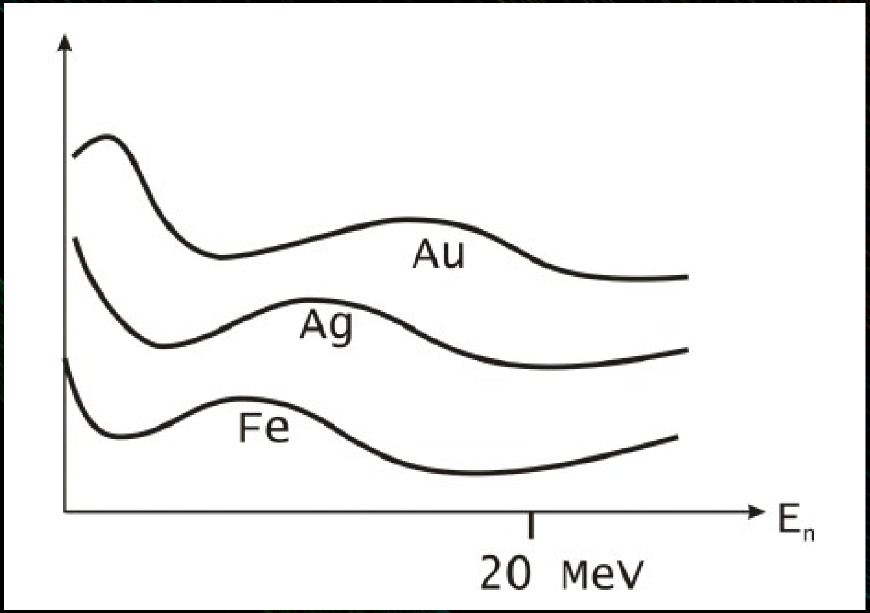
- Understanding the total neutron cross section at $E_n>1$ MeV
  - interaction between fast neutron with reduced wavelength λ and nucleus is similar as light absorption and diffraction on a black disc
  - the total cross section with this (classic limit) approximation:

$$\sigma_T=\sigma_s+\sigma_r=\pi(R+\lambdabar)^2+\pi(R+\lambdabar)^2=2\pi(R+\lambdabar)^2$$

(elastic scattering + inelastic processes)

  - good approximation: $10<E_n<50$ MeV → ±10% precision!!

- Systematic measurements show moderate amplitude oscillations, but in principle the same energy dependece for neighbouring nuclei (in contrast to the resonances which are totally different from nucleus to nucleus)
  - maxima shifts towards higher energies with A



Optical model

- Weisskopf and Feshbach proposed a reaction model to describe $\sigma_T(A,E)$
  - optical analog: light on a semi transparent medium → absorption and diffraction
  - „translating” into quantum mechanics
  - similiar to shell model description → introducing a **mean-field potential** (instead of individual nucleon-nucleon interactions)
  - the potential has a real and a complex part to deascribe scattering and absoprtion, respectively:

$$U = Vf(r) + iWg(r)$$

$$f(r) = \frac{1}{1 + e^{\frac{r-R}{a}}}$$

Woods-Saxon potential

- Solving Schrödinger equation with U → $\delta^{\pm}_l$ phase-shifts (of neutron waves in the nucleus) can be deduced → cross sections can be determined:

$$\sigma_S = \pi \lambda\!\!\!^{-}{}^2 \sum_l (2l+1)(1-\eta_l)^2$$

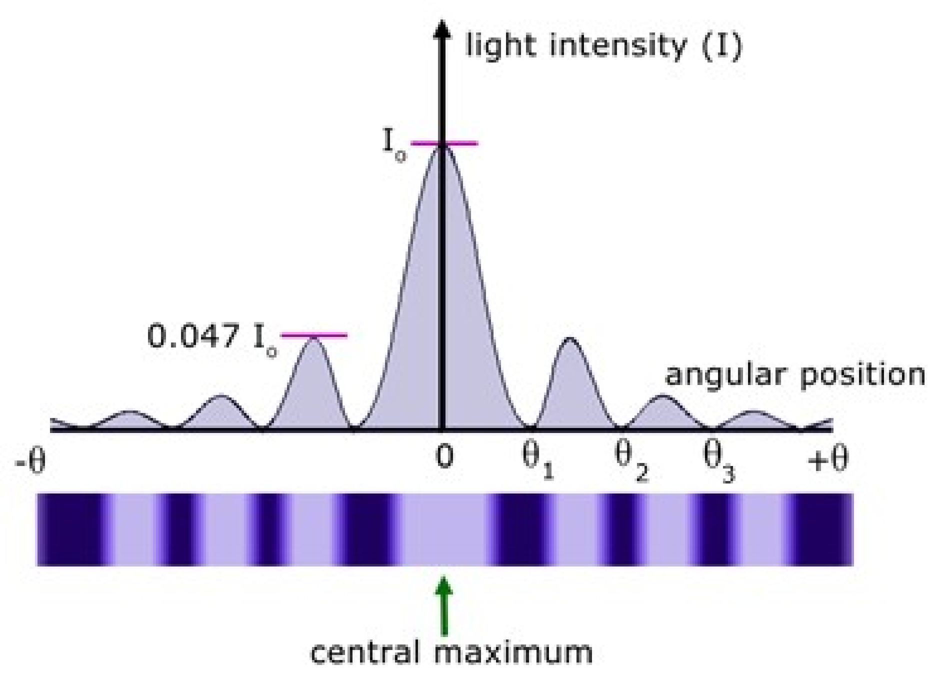
with

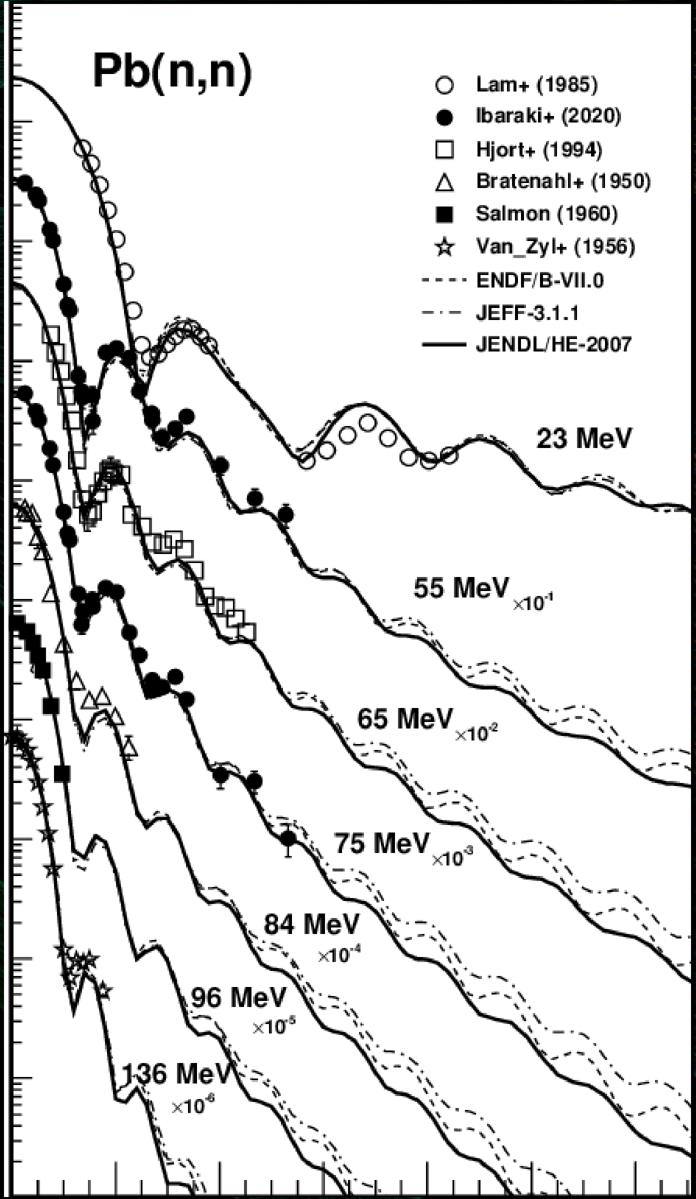
$$\eta_l^{\pm} \equiv e^{2i\delta_l^{\pm}}$$

- parameters (V, W, R, a) are fitted to the experimental cross sections
  - parameters are not varying fat → one can use these parameters to calculate experimentally unknown cross sections

Optical model

- angular distribution of neutron elastic scattering vs. light diffraction





Charged particle reactions

- Coulomb barrier + centrifugal barrier

$$B = B_{cf} + B_{Coul} = \frac{10\, l(l+1)\hbar^2}{A^{2/3}} + \frac{Zz}{A^{1/3}}$$

  - no special importance of the l=0 angular momentum transfer
- Three categories:
  - ligth charged particle induced reactions (mostly direct reactions)
  - non relativistic heavy ion reactions (very high spins can be excited)
  - relativistic heavy ion reactions
- Direct reactions:
  - Bohr compound nucleus reaction model is not explaining the energy spectrum (and angular distribution)
  - fast reaction, „direct” momentum transfer to nucleons within the nucleus
  - angular distribution is forward peaked
  - knock-out (p,n), stripping (d,p), pick-up (d,t) , break-up (d,pn) reactions
  - gives very important information on energy levels, spin, parity